Lesson 3.1.5: The Power of Exponential Growth

Targets:

1. I understand how to write an exponential formula based on a real-life scenario.
2. I understand the difference in growth between linear and exponential formulas.

Warm Up:

Two equipment rental companies have different penalty policies for returning a piece of equipment late:

- Company 1: On day 1, the penalty is $5. On day 2, the penalty is $10. On day 3, the penalty is $15. On day 4, the penalty is $20 and so on, increasing by $5 each day the equipment is late.
- Company 2: On day 1, the penalty is $0.01. On day 2, the penalty is $0.02. On day 3, the penalty is $0.04. On day 4, the penalty is $0.08 and so on, doubling in amount each additional day late.

Jim rented a digger from Company 2 because he thought it had the better late return policy. The job he was doing with the digger took longer than he expected, but it did not concern him because the late penalty seemed so reasonable. When he returned the digger 15 days late, he was shocked by the penalty fee. <u>What did he pay, and what would he have paid if he had used Company 1 instead?</u>

1. Complete the tables:

Company 1	
Day	Penalty
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

Company 2	
Day	Penalty
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

2. How much did the late fee grow for each company...
    a. Between days 5 and 6?

    b. Between days 12 and 13?

3. When does Company 2 grow faster?

4. Is the sequence Arithmetic or Geometric?
    a. Company 1:
    b. Company 2:

5. Write an explicit formula for Company 1 penalties.

6. Write an explicit formula for Company 2 penalties.

7. How much did Jim end up paying in late fees?

8. What would he have paid if he chose company 1?

Practice 1: Part 1

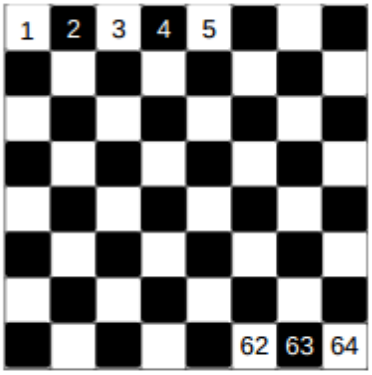
Folklore suggests that when the creator of the game of chess showed his invention to the country's ruler, the ruler was highly impressed. He was so impressed, he told the inventor to name a prize of his choice. The inventor, being rather clever, said he would take a grain of rice on the first square of the chessboard, two grains of rice on the second square of the chessboard, four on the third square, eight on the fourth square, and so on, doubling the number of grains of rice for each successive square. The ruler was surprised, even a little offended, at such a modest prize, but he ordered his treasurer to count out the rice.

a. Why is the ruler "surprised"? What makes him think the inventor requested a "modest prize"?

The treasurer took more than a week to count the rice in the ruler's store, only to notify the ruler that it would take more rice than was available in the entire kingdom. Shortly thereafter, as the story goes, the inventor became the new king.

b. Imagine the treasurer counting the needed rice for each of the 64 squares. We know that the first square is assigned a single grain of rice, and each successive square is double the number of grains of rice of the former square. The following table lists the first five assignments of grains of rice to squares on the board. How can we represent the grains of rice as exponential expressions?

Square #	Grains of Rice	Exponential Expression
1	1	
2	2	
3	4	
4	8	
5	16	



c. Write the exponential expression that describes how much rice is assigned to each of the last three squares of the board.

Square #	Exponential Expression
62	
63	
64	

Practice 1: Part 2

Use the work you completed on the previous page to help you answer these questions:

1.) Why is the base of the expression 2?

2.) What is the explicit formula for the sequence that models the number of rice grains in each square? Use n to represent the number of the square and $f(n)$ to represent the number of rice grains assigned to that square.

3.) Suppose instead that the first square did not begin with a single grain of rice but with 5 grains of rice, and then the number of grains was doubled with each successive square. Write the sequence of numbers representing the number of grains of rice for the first five squares.

4.) Suppose we wanted to represent part 3 using exponents? Would we still require the use of the powers of 2?

5.) Write an explicit formula for part 3 and compare it to the explicit formula from part 2. How are they the same? How are they different?

6.) Generalize the formula even further. Write a formula for a sequence that allows for any possible value for the number of grains of rice on the first square.

7.) Generalize the formula even further. What if instead of doubling the number of grains, we wanted to triple or quadruple them?

8.) What is the general form of an Exponential Functions? Use “a” for the starting value and “b” for the growth or decay factor.

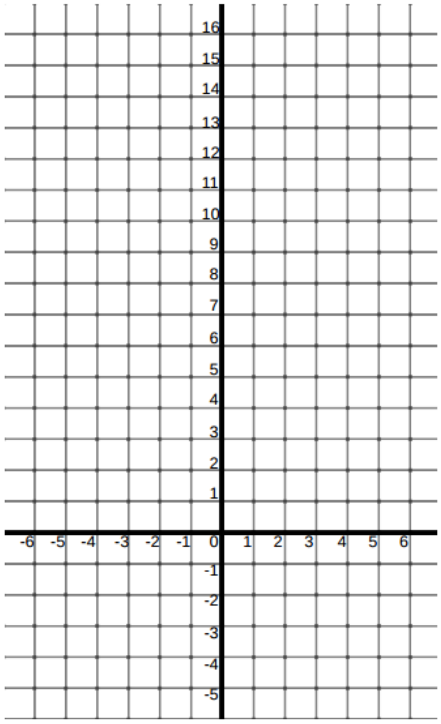
Practice 2

Let us understand the difference between $f(n) = 2n$ and $f(n) = 2^n$

a. Complete the tables below, and then graph the points $(n, f(n))$ on a coordinate plane for each of the formulas.

n	$f(n) = 2n$
-2	
-1	
0	
1	
2	
3	

n	$f(n) = 2^n$
-2	
-1	
0	
1	
2	
3	



b. Describe the change in each sequence when n increases by 1 unit for each sequence.

Practice 3

A typical thickness of toilet paper is 0.001 inches. This seems pretty thin, right? Let's see what happens when we start folding toilet paper.

a. How thick is the stack of toilet paper after 1 fold? After 2 folds? After 5 folds?

b. Write an explicit formula for the sequence that models the thickness of the folded toilet paper after n folds.

c. After how many folds will the stack of folded toilet paper pass the 1 foot mark?

d. The moon is about 240,000 miles from Earth. Compare the thickness of the toilet paper folded 50 times to the distance from Earth.

Watch the following video: "How folding paper can get you to the moon" (http://www.youtube.com/watch?v=AmFMJC45f1Q).

Practice 4

Remember back to Lesson 3.1.4 and the work we did with compound interest to help you solve this problem. A rare coin appreciates (it increases in value) at a rate of 5.2% a year, and its initial value is $500.

a. Write an exponential formula to model this situation.

b. After how many years will its value cross the $3,000 mark?

Exit Ticket

Chain emails are emails with a message suggesting you will have good luck if you forward the email on to others. Suppose a student started a chain email by sending the message to 3 friends and asking those friends to each send the same email to 3 more friends exactly 1 day after they received it.

a. Write an explicit formula for the sequence that models the number of people who will receive the email on the n th day. (Let the first day be the day the original email was sent.) Assume everyone who receives the email follows the directions.

b. Which day will be the first day that the number of people receiving the email exceeds 100?

c. How many people have received the email after 13 days? How did you calculate this?