Lesson 3.1.7: Exponential Decay

Targets:

1. I understand the difference between exponential growth and decay.
2. I understand how to represent a real situation with an exponential equation.

Warm Up:

So far we have learned about "exponential growth" which uses this equation: $f(n) = a \cdot b^n$
Here are some examples:

- The world population (in millions) was 2,519,000,000 in 1950 and grew at a rate of 1.77% each year: $f(t) = 2,519(1.0177^t)$

- The value of a house is $215,000 and its value increases at a rate of 1.5% each year: $f(n) = 215,000 \cdot 1.015^n$

- You invest $3,000 to a bank who gives you 12% interest compounded annually: $y = 3,000 \cdot 1.12^x$

What is the growth rate of the next three scenarios?

1.) You invest $500 and get _____% interest compounded annually: $y = 500 \cdot 1.08^x$

2.) A small town of 3,100 people grows at a rate of ______% each year: $f(n) = 3,100(1.011)^n$

3.) A population of 200 fruit flies grows at a rate of ______% every month: $p(m) = 200 \cdot 1.25^m$

4.) Look back at question 3. What would the equation look like if the population decreased rather than increased?

Vocabulary

1. What is "exponential decay"?

2. What is the difference between exponential growth and exponential decay?

3. Write a general equation for an exponential growth or decay formula using a for your starting value and b as your growth or decay factor.

4. What must be true about "b" for it to be "growth"?

5.) What must be true about "b" for it to be "decay"?

Introducing Decay

Mike bought a new car for \$15,000. As he drove it off the lot, his best friend, Daniel, told him that the car's value just dropped by 15% and that it would continue to depreciate 15% of its current value each year. If the car's value is now \$12,750 (according to Daniel), what will its value be after 5 years?

a. Complete the table below to determine the car's value after each of the next five years. Round each value to the nearest cent.

Number of years, t, passed since driving the car off the lot	Car value after t years	15% depreciation of current car value	Car value minus the 15% depreciation
0	\$12,750.00	\$1,912.50	\$10,837.50
1	10,837.50		
2			
3			
4			
5			

b. If the value of the car "depreciates" by 15% each year, what percentage of the value of the car is left over each year?

c. Write an explicit formula for the sequence that models the value of Mike's car t years after driving it off the lot.

d. Use the formula from part (c) to determine the value of Mike's car five years after its purchase. Round your answer to the nearest cent. Compare this value with the value in the table. Are they the same?

e. Use the formula from part (c) to determine the value of Mike's car 7 years after its purchase. Round your answer to the nearest cent

Practice 1

Identify the initial value in each formula below, and state whether the formula models exponential growth or exponential decay. **Justify your responses.**

1.) $f(t) = 2(\frac{2}{5})^t$

2.) $A(n) = 2(\frac{5}{3})^n$

3.) $y = \frac{2}{3} \cdot 3^x$

4.) $f(x) = \frac{2}{3}(\frac{1}{3})^x$

5.) $f(t) = 19,000 \cdot 1.095^t$

6.) $y = 31 \cdot 0.76^x$

Practice 2

a. If a person takes a given dosage (d) of a medication, then the formula $f(t) = d\,(0.8)^t$ represents the concentration of the medication in the bloodstream t hours later.

   i. If Charlotte takes 200 mg of the medication at 6: 00 a.m., how much remains in her bloodstream at 10 a.m.?

   ii. How long does it take for the concentration to drop below 1 mg?

b. Ryan bought a new computer for $2,100. The value of the computer decreases by 50% each year. When will the value drop below $300?

c. Kelli's mom takes a 400 mg dose of aspirin. Each hour, the amount of aspirin in a person's system decreases by about 29%. How much aspirin is left in her system after 6 hours?

Practice 3

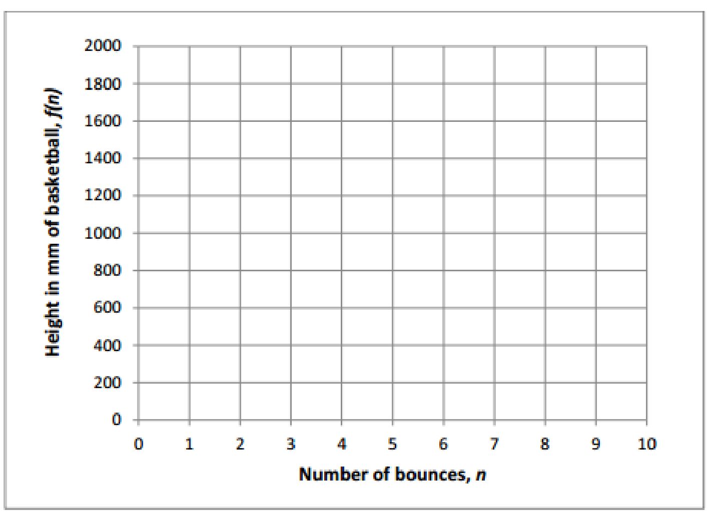
According to the International Basketball Association (FIBA), a basketball must be inflated to a pressure such that when it is dropped from a height of 1,800 mm, it will bounce to a height of 1,300 mm. Maddie decides to test the bounce-ability of her new basketball. She assumes that the ratio of each bounce height to the previous bounce height remains the same at $\frac{1300}{1800}$. Let $f(n)$ be the height of the basketball after n bounces.

a. Complete the chart below to reflect the heights Maddie expects to measure.

b. Write the explicit formula for the sequence that models the height of Maddie's basketball after any number of bounces.

n	$f(n)$
0	1,800
1	
2	
3	
4	

c. Plot the points from the table. Connect the points with a smooth curve, and then use the curve to estimate the bounce number at which the rebound height will drop below 200 mm.



Exit Ticket

A huge ping-pong tournament is held in Beijing, with 65,536 participants at the start of the tournament. Each round of the tournament eliminates half the participants.

1. If $p(r)$ represents the number of participants remaining after r rounds of play, write a formula to model the number of participants remaining.

2. Use your model to determine how many participants remain after 10 rounds of play.

3. How many rounds of play will it take to determine the champion ping-pong player?